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Scales of Gaugino Condensation and Supersymmetry Breaking in Nonstandard M-Theory Embeddings

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ABSTRACT

We investigate the formation of dynamical gaugino condensates and supersymmetry breaking in the compactifications of Horava-Witten theory with perturbative nonstandard embeddings. Specific models are considered where the underlying massless charged states of the condensing sector are determined by the spectra of $Z_2 \times Z_2$ and Z_4 orbifolds with nonstandard embeddings. We find among them viable examples where gaugino condensation is triggered on the wall with the weakest gauge coupling at M_{GUT} . In all these cases the magnitude of the condensate formed is below the energy scales at which extra dimensions are resolved, and so justifies the analysis of condensation in an effective 4-dimensional framework. We make some comments concerning the size of the largest extra dimension in the models considered. We discuss racetrack scenarios in the framework of perturbative nonstandard embeddings.

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1 Introduction

Recent advances in our understanding of the nonperturbative aspects of string theory [1] from which the idea of so called M-theory, which unifies the 5 known types of string theory has emerged, have continued to have a major impact on string inspired phenomenology. M-theory phenomenology is now a rapidly growing field which, perhaps for the first time, is allowing us to investigate detailed phenomenological questions in a consistent framework at large string coupling. There is considerable promise in models derived from strongly coupled $E_8 \times E_8$ heterotic strings which in our present understanding is realized through the Horava-Witten (H-W) construction of $d = 11$ supergravity compactified in a consistent way to $d = 10$ on S^1/Z_2 [2], [3], [4]. This model provides a natural resolution to the old puzzle that was a feature of perturbative heterotic theory, namely the consistency problems that arise if we demand that reasonable values for M_{pl} , M_{GUT} and α_{GUT} should emerge in the effective $d = 4$ theory. This resolution hinges on the fact that in the M-theory picture the effective string coupling is related to the size of the line element S^1/Z_2 which is large compared to the $d = 11$ Planck length. At the same time such a picture can naturally accommodate gauge coupling constant unification at the experimentally favoured value of 2×10^{16} GeV [3]. There is a growing body of work dealing with phenomenological aspects of compactified HW theory. Historically this began within the context of the standard embedding [5] and more recently the analysis has been extended to include nonstandard embeddings [6] - [11].¹ The correct procedure in obtaining an effective $d = 4$ action is to first integrate out the degrees of freedom on the 6-dimensional Calabi-Yau manifold and then those compactified on S^1/Z_2 [13], [14], [15], [16], [17]. This new picture provided by the H-W construction also suggests that the mechanism of supersymmetry breaking should naturally be explored in $d = 5$ rather than the usual four dimensional setting that is familiar in the weakly coupled case. This follows because consistency requires that the scale of S^1/Z_2 be hierarchically larger than the typical scale of the six dimensional compact manifold which reduces the theory from $d = 11$ to $d = 5$ [3]. The E_8 and E'_8 sectors of the four dimensional world then live on the two boundaries of this $d = 5$ supergravity model. Below the mass scale $m_5 = 1/R_5$ it is adequate to describe the low energy physics within the framework of the 4d $N = 1$ supergravity. In particular, in this framework one can study the dynamical selection of the vacuum, and low energy supersymmetry breaking. The interesting question is how the nontrivial $5d$ structure of the gauge sector, the fact that it consists of two spatially separated, although correlated through the choice of the embedding sectors, becomes reflected in the low energy effective Lagrangian. To get an insight into this problem, it is instructive to study supersymmetry breaking in simple nonstandard embedding models in the Horava-Witten setup.

Reducing from $d = 5$ to $d = 4$ around bulk field configurations that satisfy their equations of motion is a natural way of encoding the supersymmetry breaking dynamics [16], [17], [18]. For example if one considers supersymmetry breaking via the formation of gaugino condensates on a wall then one approximation is to consider these as generating a delta function source proportional to Λ^3 where Λ is the scale at which condensation on that wall. It is then possible

¹Certain aspects of nonstandard embeddings in M-theory compactifications were already discussed in [12].

in certain cases, see for instance [17], [18], to compute the size and nature of supersymmetry breaking among observable fields.

Clearly to obtain definite predictions for the scale of soft supersymmetry breaking parameters one needs to know allowed values of Λ . In this paper we want to address this question by considering a number of explicit examples in which the scales Λ may be computed. To achieve this requires knowledge of the massless charged spectrum present in the condensing sector as the latter is triggered by the running gauge coupling being driven to values larger than unity. For M-theory compactification on smooth Calabi-Yau (CY) manifolds, as in the case of weakly coupled strings it is difficult to extract explicit data concerning the massless charged spectrum in most cases. This is particularly the case of CY compactifications whose gauge and tangent bundle structure corresponds to nonstandard embeddings [19].

An alternative approach to the problem is to go to the singular limit of CY compactifications and work with orbifold compactifications. In the weak coupling limit these spaces allow the full computational power of string theory to be employed. The question arises whether the kinds of data one can compute at weak coupling such as details of the massless charged spectrum, gauge coupling threshold corrections, structure of the Kahler geometry underlying the effective low energy theory in $d = 4$ can somehow be extrapolated to the strong coupling region. In as much as one can at present only work within the consistent field theory limit of M-theory compactified on $X \times S^1/Z_2$ of the HW construction the evidence points to many of the features familiar from the weakly coupled theory surviving the extrapolation to strong coupling. An example of such features are moduli dependent gauge coupling threshold corrections. In [20] (see [21] for earlier work in the same direction) extrapolation of the latter to large string coupling by considering the large T behaviour of modular invariant threshold corrections [22], [23] results in a form of the holomorphic f function that is consistent with the one based on the supersymmetrization of Witten's threshold formula obtained from M-theory [3]. This was originally carried out for orbifolds with the standard embedding, but has recently a more general analysis has been applied to the case of nonstandard embeddings [20]. The results agree with the generalization of Witten's threshold formula to include nonstandard embeddings [6]. These extrapolations were based on modular invariant perturbative string models which match onto HW theory with nonstandard embeddings but without 5-branes in the bulk. Such extrapolations may also exist in the case where such 5-branes are included [9], [10], [11], but an explicit calculation is problematic as the corresponding weakly coupled strings are not modular invariant [20].

In this paper we consider nonstandard embeddings in HW theory without 5-branes in the bulk, where the internal compact space is taken to be an $N = 1$ orbifold. For the purpose of this paper we shall also assume that the renormalization group running of the $d = 4$ gauge couplings on each wall to be determined by charged massless states living on the wall, the latter being determined by the modular invariant orbifold construction at weak coupling.

2 Condensates on Opposite Walls

Following the reference [24] we shall summarize the main features of the racetrack models with condensates forming in two different gauge sectors, with different gauge kinetic functions.

To begin with let us, however, comment on the connection between higher (5d) and four dimensional field theory in the H-W model. This issue has been studied in detail in [24]. Here we shall recall briefly why do we expect the usual four dimensional physics to play the crucial role in supersymmetry breaking and moduli stabilization. The general equation of motion for a Z_2 -even field φ , with the sources $\mathcal{J}_v(x)$, $\mathcal{J}_h(x)$ localized on the walls and with the corresponding boundary conditions on the half-circle

$$\lim_{x^5 \rightarrow 0} \varphi' = \frac{\mathcal{J}_v}{2}, \quad \lim_{x^5 \rightarrow \pi\rho} \varphi' = -\frac{\mathcal{J}_h}{2} \quad (1)$$

has the solution of the form $\varphi(x, x^5) = \varphi(x) + \phi(x, x^5)$. In that expression $\varphi(x)$ is a truly four dimensional fluctuation, the zero mode on S^1/Z_2 , whereas ϕ is the background depending on x^5 , in the lowest order approximation given by [25, 17]

$$\phi(x, x^5) = -\frac{\mathcal{J}_v + \mathcal{J}_h}{2\pi\rho} \left(\frac{(x^5)^2}{2} - \frac{\pi\rho^2}{6} \right) + \frac{\mathcal{J}_v}{2} \left(x^5 - \frac{\pi\rho}{2} \right). \quad (2)$$

This *does not* depend on the particular form of embedding, as explained in [6], but it relies on the assumption, that the sources \mathcal{J}_v , \mathcal{J}_h vary significantly only over the scales $> \pi\rho$, see [25] for details. This condition, on which the form of the explicit solutions of the Bianchi identity relies, is nontrivial, as the sources contain not only vacuum configurations, but also include all the 4d fluctuations of fields which penetrate the walls². However, the assumption about scales is fulfilled when one is solving for the vacuum configurations, which might break supersymmetry, but should not break the 4d Lorentz invariance, so cannot depend on x^μ . At vacuum, the function $\phi(x, x^5)$ is determined in terms of the vacuum values of the sources, and perhaps by nontrivial physics in the bulk, like sigma-model interactions, but the zero mode $\varphi(x)$, with no dependence on x^5 , is left completely undetermined. This mode shall eventually be regarded as the massless field in 4d, and has to be determined by nontrivial 4d dynamics born on the walls. Let us note, that even in the schemes where bulk dynamics is credited for stabilization of some of the moduli like the radius of the fifth dimension, see [26], a nontrivial potential on the walls is necessary for stabilization³.

After this introductory comment, we can move on to the discussion of condensates, which are the obvious source of stabilizing potentials and/or hierarchical supersymmetry breaking. To be reasonably general let us start with an arbitrary number of condensates on any of the two walls (see also discussion in [27]). With 4d gauge kinetic functions of the form $f_{1,2} = S \pm \xi_0 T$

²After the averaging over the compact 6d space has been performed.

³For earlier arguments similar to that of [26] see [17]. The nontrivial warp factor used explicitly in [26] is strictly speaking a higher order correction from the point of view of [17], since boundary sources are already corrections in the expansion in powers of $\kappa^{2/3}$ (where κ is the $d = 11$ gravitational coupling).

the effective potential in this case is

$$V = \left| -\frac{e^{K/2}W}{M} + \frac{S + \bar{S}}{4M} \left(\sum_i \Lambda_i^3 \right)^2 + \frac{1}{3} \right| - 3 \frac{e^{K/2}W}{M} + \xi_0 \frac{T + \bar{T}}{4M} \left(\sum_i \epsilon_i \Lambda_i^3 \right)^2 - \frac{3}{M^2} |e^{K/2}W|^2 \quad (3)$$

where $\epsilon_i = \pm 1$ and the rest of the notation is standard and given in [24]. First, let us look for the flat space supersymmetric points of the potential (3). One reason is purely technical, namely it is much easier to find such candidate points than to look for broken supersymmetry solutions to the full equations of motion. Secondly, as we expect the realistic supersymmetry breaking scale to be hierarchically smaller than the Planck scale, one can expect the relevant points where supersymmetry is only slightly broken, to be located near the globally supersymmetric points (although the existence of remote relevant susy breaking points cannot be excluded in general). As we are looking for flat space solutions, we shall assume the vacuum expectation value of the total superpotential to be zero, which is consistent in the picture with explicit gaugino bilinears, as it does not lock to each other the values of various condensates. Then, the scalar potential takes the form

$$V = \frac{(S + \bar{S})^2}{16M^2} \left| \sum_i \Lambda_i^3 \right|^2 + \xi_0^2 \frac{(T + \bar{T})^2}{48M^2} \left| \sum_i \epsilon_i \Lambda_i^3 \right|^2 \quad (4)$$

and is equivalent to the potential obtained in the globally supersymmetric limit in the effective superpotential approach to multiple condensates. It is easy to see that existence of supersymmetric minima implies that the sum of condensates vanishes separately on each wall

$$\sum \Lambda_{i+}^3 = 0 = \sum \Lambda_{i-}^3 \quad (5)$$

at such points in field space. Here $i+, i-$ run over the number of condensates present on wall 1 and wall 2 respectively and $\{i\} = \{i+, i-\}$.

In the case of at least two condensates on each wall, equations (5) are two independent equations for two complex variables $e^{S \pm \xi_0 T}$, so generically they have a solution at finite values of S and T . The situation is such, that condensates on each wall optimize themselves and supersymmetry is unbroken, but the values of S and T become fixed. This is an interesting possibility, and it would be an interesting exercise to check which values of $Re(S)$ and $Re(T)$ can be obtained in such a setup, but in this paper we want to stay specifically within the class of calculable perturbative nonstandard embeddings discussed in [20], and there doesn't seem to be enough space for ≥ 4 condensates with realistically low condensation scales.

Hence we have to look at the vacua with 3 or less condensates. These are the interesting cases, given our comments above. First, when we want to have 3 condensates arranged on different walls, then one of them must be on one wall, and two on the opposite. Then it follows immediately that to fulfill the unbroken susy conditions the single condensate would have to vanish. With three nonvanishing condensates on both walls we therefore always have broken supersymmetry. The same applies, as pointed out in [24], to the case of two condensates on different walls.

When we have several condensates on a single wall, then this works similarly to the weakly coupled case: supersymmetry is unbroken, but only $S + \xi_0 T$ or $S - \xi_0 T$ is fixed. To have a hope

to fix both moduli within the simple, and perhaps most appealing, version of the racetrack scheme one clearly needs to consider condensates on both walls.

To conclude, in the most interesting case of two or three condensates on different walls, we can exclude the existence of flat space, unbroken supersymmetric ground states. This is not so bad however, because the existence of remote minima, disconnected from any globally supersymmetric state, cannot be excluded by a general reasoning. In fact, if one finds any proper minimum of the effective potential in these cases, supersymmetry is guaranteed to be broken there. However, to have a vanishing cosmological constant at such a minimum, one has to invoke a nonzero expectation value of the superpotential, and it would have to be of the order of Λ_{eff}^3 if we are to get the usual hierarchy $F \approx \Lambda_{eff}^3/M$. One source of such effectively constant terms in the superpotential could be condensates of the Chern-Simons forms, but then the question of the scale and ‘stiffness’ of these condensates arises, which we shall not discuss here. It is worth noting that even if the constant in the superpotential is present, then on the basis of the dynamics described by (3) one cannot tell whether it makes F_T or F_S vanish. In fact, the most plausible situation in such a case is that supersymmetry is unbroken, moduli fixed, but the cosmological constant is nonzero.

Having discussed issues of moduli stabilization, there is in addition the requirement that the condensation scale on the walls be hierarchically smaller than M_{Pl} in order to have a hope of realistic phenomenology. Of course it remains a hope that in the interesting cases mentioned above where racetrack fixing of moduli occurs, such a hierarchy of scales appears for a choice of embeddings. Whether this is possible or not, we can at least within the perturbative orbifold framework of nonstandard embeddings [20] estimate the size of condensates in some simple cases, even if we do not at the same time immediately address the issue of moduli stabilization. This will be the subject of the next section. In models where only a single T modulus exists (whose real part is proportional to the size of the 5th dimension) the form of Λ is tightly constrained if one uses as input the preferred values of α_{GUT} , M_{GUT} and M_{Pl} . If one can find reasonable values for the condensate in these models it would provide further motivation to tackling the larger problem of stabilization.

In the next Section we discuss in detail potentially realistic examples, where at the string scale, with trivial vacuum for the scalar fields, there is just one asymptotically free nonabelian factor. To repeat, the analysis is valid in the vicinity of the trivial scalar vacuum. However, in these models there are more than one nonabelian factors. In addition, usually the perturbative scalar potential has flat directions, along which some of the large nonabelian groups can be broken in such a way, that there appear additional nonabelian factors which are asymptotically free. It is straightforward to generalize in this spirit the stringy analysis we give here along the lines of [28]. This way one can easily generate models with two or three condensates.

3 Scales of Gaugino Condensation in M-theory on S^1/Z_2 : weak and strong wall cases

As was discussed in [6] one can make use of the different routes in compactifying M-theory to four dimensions, to obtain a map between "strong" and "weakly" coupled compactification moduli. This involves compactifying from $11 \rightarrow 10 \rightarrow 4$ or $11 \rightarrow 5 \rightarrow 4$. The starting metric in 11 dimensions is the same, but in the first case we can naturally express parameters of the $d = 11$ metric in terms of familiar moduli of weakly coupled 10 dimensional heterotic string theory whilst in the second we have a natural expression in terms of strong units. If for example we consider case of homogeneous Calabi-Yau compactification, then in the weak case we have $g_{\mu\nu}^{(10)} = e^{-3\sigma} g_{\mu\nu}^{(4)}$, $g_{MN}^{(10)} = e^\sigma g_{MN}^{(0)}$, $\mu = 0..3$, $M = 4..9$ with σ the breathing mode of the homogeneous CY space. Along with the dilaton ϕ this defines the real parts of weak moduli S_w and T_w as $Re(S_w) = e^{3\sigma} \phi^{-3/4}$, $Re(T_w) = e^\sigma \phi^{3/4}$. The ϕ field can be thought of as $g_{11,11}^{(11)} = \phi^2$ with a Weyl rescaling $g_{AB}^{(11)} \rightarrow \phi^{-1/4} g_{AB}^{(11)}$, $A = 0..9$ to obtain canonical gravitational action in $d = 10$. In the "strong" route to four dimensions, one identifies moduli through $g_{mn}^{(11)} = e^{-2\beta} g_{mn}^{(11)}$, $g_{MN}^{(11)} = e^\beta g_{MN}^{(11)}$ and $g_{55}^{(5)} = e^{2\gamma}$. Equating $g_{11,11}^{(11)} = g_{55}^{(5)}$ we find the relation $\phi = e^{\gamma-\beta}$. Similarly comparing $g_{MN}^{(11)}$ and $g_{MN}^{(5)}$ we obtain $e^\sigma = \phi^{1/4} e^\beta$. In this way one finds a map between "weak" and "strong" moduli which leads one to define $Re(S_s) = e^{3\beta} = Re(S_w)(\phi, \sigma)$ and $Re(T_s) = e^\gamma = Re(T_w)(\phi, \sigma)$ where the subscript s refers to strong quantities.

In the same way one can consider inhomogeneous CY spaces with different scaling moduli. In the orbifold limits these moduli are associated with the scaling modes of the underlying 6-dimensional torus as σ_i $i = 1, 2, 3$ where i labels the 3 complex planes. If we include the generalized strong moduli β_i then a similar analysis to above yields the relations $\phi = e^{\gamma - \sum_i \beta_i}$ and $e^{\sigma_i} = \phi^{1/4} e^{\beta_i}$ which leads to the definitions $Re(S_s) = e^{\sum_i \beta_i}$, $Re(T_{si}) = e^\gamma (e^{\beta_i - \frac{1}{3} \sum_i \beta_i})$ in this case. Such maps between strong and weak moduli allow one to extrapolate various weak coupling quantities to the strongly coupled regime. One may wonder if such extrapolations are consistent. In the case where one includes 5-branes in the bulk [9], [16], [11], there would appear to be problems in this respect as the corresponding weakly coupled string theory is not modular invariant [20]. If we consider the case where such 5-branes are absent (which we assume in the remainder of the paper) the answer seems more encouraging particularly regarding the moduli dependent threshold corrections. Computing these quantities in the M-theory regime confirms that they can be obtained by the maps considered above as if we additionally take the large T_i limit. The latter ensures that the moduli dependent thresholds are at most linear in the fields, which is within the Horava-Witten approximation to M-theory. Thus in what follows we will assume the perturbative results for threshold corrections in the context of N=1 orbifold compactifications and apply the above mentioned extrapolations. In what follows we shall drop the s subscript on all moduli.

We begin with a discussion of threshold effects in $E_8 \times E_8$ heterotic M-theory. In the case of smooth Calabi-Yau compactification, the difference in the unified gauge couplings α_h^{-2} and α_v^{-2} on the hidden and visible walls, for general embedding [6] can be obtained as a generalization

of the result due to Witten [3] is

$$\alpha_h^{-1} - \alpha_v^{-1} = -\frac{s_i}{4\pi^2} T(n_{Fi} - \frac{1}{2}n_R) \quad (6)$$

where the single $T = e^\gamma$ modulus appears if we consider only the overall $(1, 1)$ modulus of the CY manifold. The instanton numbers n_{Fi} and n_R are defined as

$$\int_X \omega \wedge \text{tr}(F^{(i)} \wedge F^{(i)}) = -\frac{1}{2} \int_X \text{tr} F_{ij}^{(i)} F^{(i)ij} = -4\pi^2 n_{Fi} \leq 0 \quad (7)$$

and

$$\int_X \omega \wedge \text{tr}(R \wedge R) = -\frac{1}{2} \int_X \text{tr}(R_{ij} R^{ij}) = -4\pi^2 n_R \leq 0 \quad (8)$$

The integrability conditions for the equations of motion give constraint on the instanton numbers⁴ [30]

$$n_{F1} + n_{F2} = n_R \quad (9)$$

Let us take standard embedding first. There $n_{Fv} = n_R$, and

$$\alpha_v^{-1} - \alpha_h^{-1} = \frac{1}{32\pi^3} T n_R \quad (10)$$

This particular embedding gives the specific gauge group structure $E_{6(v)} \times E_{8(h)}$. We stress that since n_R has positive sign, there is no way of changing the sign of $\alpha_v^{-1} - \alpha_h^{-1}$ without going to an entirely different gauge group structure, which means going to a new, different from the standard one, embedding. In the S and T notation we have ($\alpha' = 1/2$)

$$\alpha_v^{-1} = 4\pi(S + \epsilon T), \quad \alpha_h^{-1} = 4\pi(S - \epsilon T), \quad \epsilon = \frac{n_{Fv} - \frac{1}{2}n_R}{32\pi^3} \quad (11)$$

where $S = 1/(g_4^2) - i\frac{\theta}{8\pi^2}$ and $T = r + i\Sigma$ are defined such that the correct axionic shift invariances of the low energy $d = 4$ action, in the presence of instantons, is $\theta \rightarrow \theta + 2n\pi$ and $\Sigma \rightarrow \Sigma + 16\pi^2$. Eqn(11) holds for general embeddings. The labels v and h pertain to the standard embedding case where standard model gauge group and matter representations are associated with the E_6 sector. However it may well be that for certain embeddings the standard model matter might emerge from either wall, thus leading to the possibility that visible matter has strongest gauge coupling at M_{GUT} (explicit examples are given later) We continue to use the labeling v and h in the general case but the latter point should be borne in mind.

The above threshold formulae correspond to following gauge coupling functions

$$f_h = S - \frac{(n_{Fv} - \frac{1}{2}n_R)}{32\pi^3} T, \quad f_v = S + \frac{(n_{Fv} - \frac{1}{2}n_R)}{32\pi^3} T \quad (12)$$

So far these thresholds apply in the case of smooth CY spaces. We are interested in this paper in the orbifold limit, in which case we need the analogue of eq(7), (8). In [20] this

⁴In fact, the configurations we use here fulfill the Yang-Mills equations of motion and Einstein equations which justifies the term instanton.

problem was addressed within the context of nonstandard embeddings and the M-theory limit. It was argued that topological integrals in eqs(7),(8) would only be nonvanishing if there are codimension 2 fixed points appearing. From this it was argued that effectively dynamics of $K_3 \times T^2$ compactification controls the non-zero SU_2 instanton numbers. The orbifold limit then corresponds to taking K_3 orbifolds T^4/G where G is an appropriate discrete group. The computation of allowed instanton numbers of gauge and tangent bundles over K_3 is simplified if the point like singularities of T^4/G are repaired by so called ALE spaces [20]. In this case the SU_2 instanton numbers $n_i^{(1)}, n_i^{(2)}$ are given by

$$\int_{C_i} Tr(F^{(1)} \wedge F^{(1)}) = n_i^{(1)} \quad \int_{C_i} Tr(F^{(2)} \wedge F^{(2)}) = n_i^{(2)} \quad (13)$$

where C_i are 4-cycles associated with codimension 2 fixed points labelled by $i = 1 \dots h_{1,1}$ defined so that $\int_{C_i} d^j = \delta_i^j$ and $\int_X J_i \wedge d^j = \delta_i^j$ where d^i are basis of harmonic $(2,2)$ forms on the CY space X and J_i a basis for $(1,1)$ forms. One can further express $n_i^{(1)}, n_i^{(2)}$ in terms of the orbifold data such as order of fixed points ν_α and shifts γ_α^I defined by the $N = 2$ embedding [20]. Because of the $K_3 \times T^2$ structure, the Bianchi identities imply that $n_i^{(1)} + n_i^{(2)} = \chi_{K_3} = 24$. Thus in going to the orbifold limit of CY spaces we are forced to consider a more restrictive set of instanton numbers than in the smooth case. The corresponding expressions for the M-theory threshold corrections, analogous to eq.(12) will be [20]

$$f_h = S - \frac{1}{32\pi^3} \sum_i \frac{|D_i|}{|D|} (n_i^{(1)} - 12) T^i \quad , \quad f_v = S + \frac{1}{32\pi^3} \sum_i \frac{|D_i|}{|D|} (n_i^{(2)} - 12) T^i \quad (14)$$

The observation that it is $K_3 \times T^2$ dynamics that controls the thresholds corrections via the SU_2 instanton numbers is completely consistent with the perturbative picture of threshold corrections which are also associated with the existence of $N = 2$ supersymmetric fixed planes [22], [29] .

Let us contrast the above threshold corrections to the following exact form of the holomorphic couplings in the large T^i limit of orbifold compactification in $E_8 \times E_8$ heterotic string [22]. (This is obtained assuming general case of $h_{1,1}$ different T moduli and general levels k_v, k_h and assuming our present notation for sectors labelled by "h" and "v"):

$$f_h = k_h \tilde{S} + \frac{1}{48\pi} \sum_i \gamma_h^i \tilde{T}^i \quad ; \quad f_v = k_v \tilde{S} + \frac{1}{48\pi} \sum_i \gamma_v^i \tilde{T}^i \quad (15)$$

where

$$\gamma_a^i = \sum_I T_a(R_I) (1 + 2n_I^i) - C(G_a) \quad (16)$$

where I runs over the dimension of the matter field irrep R_I of group factor G_a . $T_a(R_I) = Tr_{R_I}((t^a)^2)$ for any generator t^a of G_a . $C(G_a)$ is the same quantity but with R_I given by adjoint representation. The numbers n_I^i define the so called modular weights of matter fields

transforming in R_I irrep, with respect to T^i duality transformations. (Note n_I^i is the negative of the modular weights q_I^i defined in [22])

As was pointed out in [22] , there is another expression for the coefficients γ_a^i whose validity ensures that the field and string theoretical threshold calculations agree. This has the form (for general levels k_h and k_v :

$$\gamma_a^i = \frac{b_a^{N=2}(i)}{|D|/|D_i|} + k_a \delta_{GS}^i \quad (17)$$

where in this formula, $b_a^{N=2}(i)$ is the $N = 2$ beta function associated with massless spectrum originating from the twisted sector having the invariant plane labelled by i . D is the orbifold point group and D_i the little group corresponding to the invariant plane.

Using both these forms of γ_a^i we can derive an expression for the $N = 1$ beta function $b_a = -3C(G_a) + \sum_I T_a(R_I)$ that will be useful in the formula for the scale of gaugino condensation.

$$\frac{1}{3}b_a = \frac{b_a^{N=2}(i)}{|D|/|D_i|} - \sum_I T_a(R_I)(2/3 + 2n_I^i) + k_a \delta_{GS}^i \quad (18)$$

The definition of the T moduli are such that $T^i = r^i + i\theta^i$ with the axionic shift invariance given by $\theta^i \rightarrow \theta^i - 4n^i$, n^i integers. Notice that in this basis the T^i dependent thresholds do not just differ by opposite signs. However by redefining the \tilde{S} field to a new field S' through T^i dependent shifts one can always bring these into the form directly comparable with eq(14) (but for generic levels k_h, k_v)⁵

$$f_h = k_h S' + \frac{1}{96\pi} \sum_i (\gamma_h^i - \gamma_v^i) \tilde{T}^i \quad , \quad f_v = k_v S' - \frac{1}{96\pi} \sum_i (\gamma_h^i - \gamma_v^i) \tilde{T}^i \quad (19)$$

By comparing the latter formula with the previous M-theory thresholds in the case of nonstandard embeddings, and taking into account the different normalizations of T moduli we find that in the orbifold limit, and for a single T modulus (labelled by some fixed value of i)

$$n_i^{(1)} - 12 = \frac{|D|}{12|D_i|} (\gamma_v^i - \gamma_h^i) \quad (20)$$

where in eqn(20) the v and h subscripts generically refer to the *visible* or weaker sector and the hidden or stronger sector respectively. If these sectors are direct products of simple groups then there is a corresponding value of γ_a^i for each such group factor.

Now in the orbifold limit, expressions for $b_v^{N=2}$ and $b_h^{N=2}$ has been obtained in [20] by extrapolation of weak coupling expressions to the M-theory domain. These depend on instanton numbers of gauge field configurations over K_3 since in the orbifold limit the threshold corrections are essentially based on $K_3 \times T^2$ dynamics even though the full theory involves

⁵In the basis where S is not shifted, there are additional holomorphic, universal corrections to f_h and f_v that yield the antisymmetric nature of the threshold corrections [23], [20]

compactification on a Calabi-Yau manifold giving $N = 1$ supersymmetry in $d = 4$. Defining $n_i^{(1)} = 12 - n_i$ and $n_i^{(2)} = 12 + n_i$ the expressions for the $N = 2$ beta functions are [20]

$$b_v^{N=2} = 12 - 6n_i \quad , \quad b_h^{N=2} = 12 + 6n_i \quad (21)$$

(here we assume no Wilson line breaking).

Note that there is no explicit group index appearing in the above definitions, so that if the unbroken gauge group in the four dimensional effective theory is indeed a product of simple factors (in either the weak or strong sector) then the expressions for the $N = 2$ beta functions must coincide for each group factor. This can be seen in a number of explicit examples discussed in [20] and also is implicit in the nonstandard embedding orbifold examples in [22] . The holomorphic f functions can be written as

$$\begin{aligned} f_h &= k_h S + \frac{1}{32\pi^3} \left(\frac{|D|}{|D^i|} n_i + (k_v - k_h) \delta_{GS}^i \right) T^i \\ f_v &= k_v S - \frac{1}{32\pi^3} \left(\frac{|D|}{|D^i|} n_i + (k_v - k_h) \delta_{GS}^i \right) T^i \end{aligned} \quad (22)$$

where in these equations we have returned to the definition of S and T^i moduli with the normalizations given earlier.

Before we consider the specific scale of gaugino condensation, we can write down a general form for the $N = 1$ beta functions that will appear in the expression for gaugino condensates, using previous relations, again in the extrapolated orbifold limit:

$$\begin{aligned} b_v &= 3 \frac{|D^i|}{|D|} (12 - 6n_i) - 3 \sum_I T_v(R_I) (2/3 + 2n_I^i) + 3k_v \delta_{GS}^i \\ b_h &= 3 \frac{|D^i|}{|D|} (12 + 6n_i) - 3 \sum_I T_h(R'_I) (2/3 + 2n'^i_{I'}) + 3k_h \delta_{GS}^i \end{aligned} \quad (23)$$

where the apostrophe ' distinguishes matter representations coming from hidden sector. Again we emphasise that in these formulae, the i labels a single fixed plane under the action of the orbifold point group.

We can now write the following general expressions for the v and h sector gaugino condensates Λ_v, Λ_h . In the case of v sector condensation, (we set $k_v = k_h = 1$ for simplicity)

$$\Lambda_v = M_{GUT} \left(\frac{\alpha_{GUT}^{-1}}{\alpha_{GUT}^{-1} + 8\pi\epsilon T_r} \right)^{\frac{1}{6}} e^{\frac{2\pi}{b_v} (\alpha_{GUT}^{-1} + 8\pi\epsilon T_r)} \quad (24)$$

and in the case of h sector condensation,

$$\Lambda_h = M_{GUT} \left(\frac{\alpha_{GUT}^{-1}}{\alpha_{GUT}^{-1} - 8\pi\epsilon T_r} \right)^{\frac{1}{6}} e^{\frac{2\pi}{b_h}(\alpha_{GUT}^{-1} - 8\pi\epsilon T_r)} \quad (25)$$

where in both these expressions $\epsilon = \frac{1}{32\pi^3} \left(\frac{|D|}{|D^i|} n_i \right)$. In the examples based on perturbative non-standard embeddings in orbifolds which we discuss below, it turns out that condensation occurs on only one wall at a time. As was mentioned in the last section, while this is not the most interesting case from the point of view of complete dynamical fixing of moduli, it will provide a framework in which to estimate the scale of soft susy breaking. This simplification of single wall condensation means that the expression for the observable inverse gauge coupling $\alpha_{GUT}^{-1} = 4\pi(S_r - \epsilon T_r)$ when condensation occurs on the weaker wall in the v sector, whereas if it occurs on the strong wall $\alpha_{GUT}^{-1} = 4\pi(S_r + \epsilon T_r)$. Of course in either case the triggering of condensation requires the particularly sector to contain gauge and massless matter fields that lead to an asymptotically free theory. This requires a negative b_v or b_h and we shall discuss these conditions shortly.

Now we note that the lowest order (in $\kappa^{2/3}$)⁶ fit to the 4-dimensional Planck mass M_{pl} and GUT scale M_{GUT} gives

$$T_r = \frac{M_{pl}^2 (\alpha_{GUT})^{\frac{1}{3}}}{2^{\frac{17}{3}} \pi^3 M_{GUT}^2}$$

$$M_{GUT}^{-6} = 2\pi \text{Re}S (4\pi\kappa^2)^{2/3} \quad (26)$$

which shows that the vacuum expectation value of T_r is not a free parameter. From this constraint it can be seen that in general the scale of the condensate is given as a function of $M_{pl}, M_{GUT}, \alpha_{GUT}, n_i, \delta_{GS}^i, k_v, k_h$ as well as the matter field modular weights $n_I^i, n_{I'}^i$ and Dynkin labels $T_v(R_I), T_h(R_{I'})$. Note that if we take $M_{pl}, M_{GUT}, \alpha_{GUT}$ as more or less fixed by the MSSM (at this point we put aside the possibility of so called strong unification [31]), then the remaining parameters depend on the specific details of choice of $N = 1, d = 4$ orbifold, along with choice of embedding, Wilson lines etc.

Before looking at some detailed models, it is worth investigating in the orbifold limit of Horava-Witten M-theory, if one can realize the scenario discussed in [6] that condensation can occur on either of the two walls, with corresponding observable sectors on opposite wall. Firstly let us note that in the weakly coupled heterotic compactifications, there appears somewhat different constraint on the allowed values of modular weights $n_I^i, n_{I'}^i$ depending on whether one considers large volume smooth Calabi-Yau or singular orbifold compactifications. In the former [32] it has been noted that calculations of matter field contributions to the Kahler potential show that in this case, in known examples, the modular weights are $n_I^i = n_{I'}^i = -1/3$, which is characteristic of matter fields arising from the untwisted sectors of orbifolds. These values imply that in the expressions for the $N = 1$ beta functions given above, the terms corresponding to matter irreps drop out, and one finds

⁶Where κ is the $d = 11$ gravitational coupling.

$$b_v = 3 \frac{|D^i|}{|D|} (12 - 6n_i) + 3k_v \delta_{GS}^i \quad , \quad b_h = 3 \frac{|D^i|}{|D|} (12 + 6n_i) + 3k_h \delta_{GS}^i \quad (27)$$

where we have substituted for orbifold expressions for the $N=2$ beta functions in each sector, since as argued in [20], the value of these quantities is not expected to change in going from CY to the orbifold limit. Thus in the cases where $\delta_{GS}^i = 0$, we find that there is quite a strong constraint on the allowed values of the beta functions. Recall that in our choice of embeddings, $n_i < 0$ (with the standard embedding corresponding to $n_i = -12$), so that it is clear that condensation can only occur in the h sector under these conditions. If we relax $\delta_{GS}^i = 0$ then since k_v, k_h are both positive integers (by assumption that the corresponding affine algebras are unitary) the only possibility to realize condensation in v sector is to take $\delta_{GS}^i < 0$.

Even then, we have to ensure that even if δ_{GS}^i is sufficiently negative to turn b_v negative, it must not at the same time turn b_h negative since it is the h sector where we identify with the observable sector, e.g. the MSSM where we know that the corresponding beta functions are positive. Thus it is clear that some kind of hierarchy between the levels k_v and k_h is needed if this mechanism is to work, the question is how large should this be? and in addition how negative should δ_{GS}^i be? Evidently we need to constrain $|n_i|$ so that the terms independent of δ_{GS}^i in both beta functions is positive, which requires $|n_i| < 2$ or since the n_i are integer and negative, we are lead to specifically $n_i = -1$. We then find the inequalities $|k_h \delta_{GS}^i| < 6 \frac{|D^i|}{|D|}$ and $|k_v \delta_{GS}^i| > 18 \frac{|D^i|}{|D|}$ to realize $b_v < 0, b_h > 0$ i.e. $\frac{6}{k_h} > |\delta_{GS}^i| > \frac{18}{k_v}$. This requires $k_v > 3k_h$. Although it may seem that one may have arbitrarily large values of k_v, k_h in fact from the formulae given earlier for δ_{GS}^i we see that the latter are integer valued, which clearly bounds allowable values of the levels.

The expression for the v sector condensate appropriate to this case, which is a generalization of that given earlier to the case where k_v, k_h are different from unity, is

$$\Lambda_v = M_{GUT} \left(\frac{\alpha_{GUT}^{-1}}{\frac{k_v}{k_h} \alpha_{GUT}^{-1} + \frac{1}{4\pi^2} \text{Tr}(1 + \frac{k_v}{k_h})} \right)^{\frac{1}{6}} e^{\left(\frac{\pi(\frac{k_v}{k_h} \alpha_{GUT}^{-1} + \frac{1}{4\pi^2} \text{Tr}(1 + \frac{k_v}{k_h}))}{27 + \frac{3k_v \delta_{GS}^i}{2}} \right)} \quad (28)$$

where in the above we have substituted for $\epsilon = \frac{1}{32\pi^3}$. By examining the scale Λ_v as a function of α_{GUT} and δ_{GS} for various values of k_v, k_h satisfying $k_v > 3k_h$ one can see that reasonable values may be obtained but the question remains if one can easily find explicit models with these values of k_v, k_h and δ_{GS} .

Having discussed the case relevant to large volume Calabi-Yau compactification with all modular weights $n_I^i = n_{I'}^{i'} = -1/3$ we want to consider compactifications on $N = 1$ orbifolds where more general values of the modular weights occur. This means that there is a weaker connection between the $N = 2$ and $N = 1$ beta functions since now matter representations contribute to the right hand side of eq(23). Therefore we are forced to consider explicit orbifold models, where the full $N = 1$ charged, massless spectrum is known in which to evaluate the size of the gaugino condensate. In this paper we look at all the nonstandard embeddings in $Z_2 \times Z_2$

and Z_4 orbifolds (here we do not consider Wilson lines) and consider what gauge sectors in each model can support gaugino condensation. Also we shall determine (if condensation does occur), whether it falls into the class of "weaker" wall or "stronger" wall condensation in the sense defined earlier. Such a possibility is a consequence of nonstandard embeddings. First we consider $Z_2 \times Z_2$ models. There are 5 modular invariant embeddings of the point group into $E_8 \times E_8$ including the standard embedding giving the unbroken gauge group $E_6 \times U_1^2 \times E_8'$. Details of these models together with the full charged massless spectrum is given in [22]. The gauge coupling threshold corrections in this case will depend on all three $T_i, i = 1, ..3$ since each complex plane has a nontrivial little group among the $Z_2 \times Z_2$ generators. Moreover since each such little group defines a Z_2 , one also has three U-moduli U_i , which the threshold corrections also depend on (infact in a way which respects T_i, U_i exchange symmetry [22]) As usual the T -moduli are associated with deformation of Kahler class whilst U_i modify the complex structure. In determining the allowed values of the gaugino condensate, we have to take into account the constraint that connects M_{pl} with M_{GUT}, α_{GUT} and other parameters appearing in the M-theory compactification to $d = 4$. In the case of several T_i moduli, the formula for M_{pl} is generalized to

$$\left(\prod_i Re(T_i)\right)^{1/3} = \frac{M_{pl}^2 (\alpha_{GUT})^{\frac{1}{3}}}{2^{\frac{17}{3}} \pi^3 M_{GUT}^2} \quad (29)$$

which fixes the "overall" modulus $(\prod_i Re(T_i))^{1/3}$ in terms of M_{GUT}, M_{pl} and α_{GUT} . Here M_{GUT} is as given in eq(26) with the definition of S generalized to the inhomogeneous CY case as discussed earlier. But this still leaves a certain freedom in the precise values of T_i . We shall see in a moment that certain values of T_i might be favoured over others. Note that this relation does not involve the U_i so its dynamics are less constrained by realistic values for M_{pl}, M_{gut} etc.. Although the threshold corrections do depend on U_i we shall set $U_i = 0, i = 1, ..3$ in what follows and comment on the significance of non-zero U_i at the end. Table 1 shows $Z_2 \times Z_2$ models with nonstandard embeddings, and corresponding gauge groups. Note that only 2 of these have sectors which give negative $N = 1$ beta function when one includes all charged massless matter listed in [22] and hence allow condensation to occur. Table 2 also lists the corresponding value of the instanton numbers n_i corresponding to each $N = 2$ plane, and which dictate the explicit moduli dependence of the gauge coupling threshold corrections (eq.(22))

As discussed in [22] there are basically two distinct $N = 2$ gauge groups associated with the possible $N = 2$ planes, namely $E_7 \times SU_2 \times E_8'$ or $E_7 \times SU_2 \times SO'_{16}$ with $N = 2$ massless matter content described in [22]. The surviving $N = 1$ gauge group, obtained after GSO projection will be a subgroup of either of these. Note in $Z_2 \times Z_2$ the GS coefficients $\delta_{GS}^i = 0$ and we take levels k_h and k_v for all non-abelian factors to be unity. Also in this case $|D|/|D_i| = 2$ for all planes. Although in principle the condensate Λ can depend on all three real moduli ReT_i , for now we simplify matters by considering its dependence in the direction $T_1 = T_2 = a T_3 \equiv a T$ where a is some positive real parameter. Then effectively Λ depends on 2 of the three real parameters, a and ReT . The relation eq(29) will fix ReT so that Λ is naturally a function of the scale a and α_{GUT} assuming M_{pl} is fixed. We will comment on more general case later.

The expressions for the f functions and condensate scale Λ in the condensing models 1 and

3 of table 1 (along this direction in moduli space) are:

Model 1

$$\begin{aligned}
f_{E_6} &= S - \frac{(2a+1)}{16\pi^3} T, & f_{SO'_8} &= S + \frac{(2a+1)}{16\pi^3} T \\
\Lambda_{E_6} &= M_{GUT} \left(1 - \left(\frac{(2a+1) M_{pl}^2 \alpha_{GUT}^{2/3}}{M_{GUT}^2 32\pi^4 2^{5/3} a^{2/3}} \right) \right)^{-1/6} e^{-\frac{\pi}{9} [\alpha_{GUT}^{-1} - \frac{(2a+1)}{2\pi^2} (\frac{M_{pl}^2 \alpha_{GUT}^{-1/3}}{M_{GUT}^2 16\pi^3 2^{5/3} a^{2/3}})]} \quad (30)
\end{aligned}$$

Model 3

$$\begin{aligned}
f_{E_7} &= S - \frac{(2a+3)}{16\pi^3} T, & f_{SU'_8} &= S + \frac{(2a+3)}{16\pi^3} T \\
\Lambda_{E_7} &= M_{GUT} \left(1 - \left(\frac{(2a+3) M_{pl}^2 \alpha_{GUT}^{2/3}}{M_{GUT}^2 32\pi^4 2^{5/3} a^{2/3}} \right) \right)^{1/6} e^{-\frac{\pi}{21} [\alpha_{GUT}^{-1} - \frac{(2a+3)}{2\pi^2} (\frac{M_{pl}^2 \alpha_{GUT}^{-1/3}}{M_{GUT}^2 16\pi^3 2^{5/3} a^{2/3}})]} \quad (31)
\end{aligned}$$

In obtaining these expressions we have solved the constraint eq(29) for T . Note that in both cases the condensing sector has the strongest coupling of the two walls at the GUT scale, which can be seen from the relative minus signs in the corresponding threshold corrections.

In Figures 1-2 we have plotted Λ as a function of the scale a and α_{GUT} , where the range of values of α_{GUT} are chosen near the MSSM preferred value of 0.04. What is particularly interesting about both plots is that they show that the condensate has a dependence on a which has a distinct minimum. This suggests that the dynamics of the condensate might be such that it would adjust itself to favour a close to its minimum. Strictly speaking one should minimize not Λ but rather the effective potential V given in the Section 2. However one can check that the position of the minima in T_i space is only shifted slightly, with the value of a at the minimum remaining unchanged, so Figures 1-2 are qualitatively correct. In model 1, the minimum is close to $a = 1$ which corresponds to the "overall modulus" direction $T^1 = T^2 = T^3$ in moduli space. The condensate in Model 2 shows a minimum close to $a = 3$. The value of Λ/M_{pl} is seen to be about 10^{-5} for model 1 and 7×10^{-4} in model 3, again assuming values of α_{GUT} close to 0.04. The scale of Λ in the latter case might seem to be too high for intermediate scale supersymmetry breaking, but infact in this case Λ is very sensitive to the value of α_{GUT} chosen.

For example choosing $\alpha_{GUT} = 0.02$ gives $\Lambda \approx 5 \times 10^{-6} M_{pl}$. Moving on to consider more generic points in the T^i moduli space, it turns out that there is a global minimum of Λ_v as a function of $Re(T^2), Re(T^3)$ (solving the constraint eq(29) in terms of $Re(T^1)$). These lie approximately along the directions $T^1 = T^2 = T^3$ and $T^1 = T^2 = 3T^3$ for Models 1 and 3

respectively, as can be seen in Figures 3-4. Hence studying the condensate along such directions is natural.

Let us now move on to discuss the case of nonstandard embeddings in Z_4 orbifolds. Again if we do not consider Wilson lines, there are 12 inequivalent modular invariant embeddings of the Z_4 point group generated by $\Theta = (i, i, -1)$ into $E_8 \times E_8$ (e.g. see [33]). As usual the moduli dependent threshold corrections depend on the $N = 2$ invariant planes, which in this case corresponds to the $i = 3$ plane which has little group $D \equiv Z_2$. As is argued in [22] this fact implies that the GS coefficients δ_{GS}^i are vanishing in all Z_4 orbifolds as indeed they are for the $Z_2 \times Z_2$ models discussed above. The only nonvanishing instanton numbers are n_3 and are determined by the $N = 2$ gauge group and spectra corresponding to this fixed plane. Since the little group is again Z_2 there are just two possible $N = 2$ supersymmetric sectors which we label type A and B. Type A has gauge group $E_7 \times SU_2 \times E'_8$ and massless charged fields consisting of $2 \times (\mathbf{56}, \mathbf{2}; \mathbf{1}) \oplus 16 \times (\mathbf{56}, \mathbf{1}; \mathbf{1}) \oplus 64 \times (\mathbf{1}, \mathbf{2}; \mathbf{1})$. Type B has gauge group $E_7 \times SU_2 \times SO'_{16}$ and massless charged fields consisting of $2 \times (\mathbf{56}, \mathbf{2}; \mathbf{1}) \oplus 2 \times (\mathbf{1}, \mathbf{1}; \mathbf{128}) \oplus 16 \times (\mathbf{1}, \mathbf{2}; \mathbf{16})$. In addition there are of course the massless vector multiplets that can, after the full projection that breaks $N = 2$ to $N = 1$, give rise to charged massless chiral multiplets. All this means that in $N = 1$ models based on type A, $N = 2$ sectors, the instanton number is $n_3 = -12$ and those based on type B have $n_3 = 4$. Table 2 lists the allowed nonstandard embeddings, and in each case one can determine by inspection which of the models type A or B they are associated with. To determine the exact embedding of the $N = 1$ gauge group into $N = 2$ requires comparing the $N = 1$ massless spectra with the $N = 2$ spectra listed above ⁷. This identification is important in deducing whether the condensing sector is the more strongly or weakly coupled, a fact which has important consequences that we discuss later. For example in the case of model 1, we have $N = 1$ gauge group $E_6 \times SU_2 \times E'_7 \times SU'_2$ which is naturally associated with type A, $N = 2$ gauge group. The threshold corrections relevant in the M-theory region are therefore (we define $T^{i=3} = T$)

$$\begin{aligned} f_{E_6} &= S + \frac{3}{16\pi^3}T = f_{SU_2} \\ f_{E'_7} &= S - \frac{3}{16\pi^3}T, \quad f_{SU_2} = S - \frac{3}{16\pi^3}T \end{aligned} \tag{32}$$

which shows that E_6 and SU_2 gauge sectors are more weakly coupled and in this example gauginos of the E'_7 sector condense.

An example of an $N = 1$ spectra based on type B, $N = 2$ gauge group is in model2 of table 2. Here we find

$$f_{E_6} = S - \frac{1}{16\pi^3}T = f_{SU_2}$$

⁷For brevity we have not listed the various $N = 1$ charged massless states of the models listed in tables 1 and 2.

$$f_{SO'_{16}} = S + \frac{1}{16\pi^3} T \quad (33)$$

Thus in this case the SO'_{16} is the more weakly coupled (at M_{GUT}) and is also the sector where condensation occurs. In a similar manner one can derive the threshold corrections for the models listed in table 2 and determine the weaker/stronger coupled sector.

In order to compute the $N = 1$ beta functions with a view to discovering which models can support gaugino condensation, it is necessary to have knowledge of the complete charged massless spectrum. In [34] a classification of the \mathcal{T}_k twisted sectors of Z_{2k} , $N = 1$ orbifolds was presented. In the present case of Z_4 [33] list the massless states from the \mathcal{T}_1 and \mathcal{T}_2 twisted sectors. To complete the analysis we have just to consider if any additional massless states can arise from the twisted sector \mathcal{T}_3 ($k = 0..3$, with \mathcal{T}_0 corresponding to the untwisted sector in the Z_4 case). The equations defining massless right and left-moving states are

$$\begin{aligned} \frac{1}{2} \sum_{t=1}^4 (p^t + k v^t)^2 + N_R^{(k)} + c_k - \frac{1}{2} &= 0 \\ \frac{1}{2} \sum_{I=1}^{16} (P^I + k V^I)^2 + N_L^{(k)} + N_{E_8 \times E'_8} + c_k - 1 &= 0 \end{aligned} \quad (34)$$

where $N_{R,L}^{(k)}$ and $N_{E_8 \times E'_8}$ are oscillator numbers in the right and left moving degrees

The point group element Θ acting on the $SO(8)$ bosonized fermion lattice is represented by the shift vector given by $v = \frac{1}{4}(1, 1, -2, 0)$. The normal ordering constants c_k are $c_1 = c_3 = 5/16$ and $c_2 = 1/4$. Now since $\min((p + 3v)^2) = 22/16$ (where p is an $SO(8)$ lattice vector) it is clear that there are no additional massless modes coming from the \mathcal{T}_3 twisted sector. (In the \mathcal{T}_2 sector for example there are massless states since now $\min((p + 2v)^2) = 1/4$. Thus the end result is that we can use the states given in [33] to determine the $N = 1$ beta functions in Z_4 models and those which have negative values, and which are therefore relevant in gaugino condensation are listed in table 2. From our above discussion of stronger and weaker coupled sectors at M_{GUT} , it turns out that all of the models that support a condensing sector, this sector is the more strongly coupled except in two cases: models 2, 4 and 9 where this occurs in the weaker sector. Since in the Z_4 case there is only one T modulus contributing to thresholds, the size of the condensate (either Λ_v or Λ_h as appropriate) is only a function of α_{GUT} (for fixed M_{pl} and M_{GUT}).

Before closing it is worth while considering what the largest allowable value for the radius of the 5th dimension can be in the models considered in this paper, consistent with the various constraints on low energy couplings etc that we have seen above. There has been much recent interest in models that macroscopically large (wrt e.g. the GUT scale 2×10^{16} GeV) extra spatial dimensions.

One of the basic constraints related to the maximum physical size of the 5th dimension, $\pi\rho = \pi \text{Re}(T)\rho_0$ ⁸ and the choice of whether the observable sector lies on the wall that is more

⁸In [6] we argued that a natural choice for the reference scale ρ_0 is through $\alpha' = \frac{1}{(4\pi)^{2/3}\pi^2} \frac{\kappa^{2/3}}{\rho_0}$

weakly/strongly coupled. In the former case we have the relation $\alpha_{GUT} = \text{Re}S + \epsilon \text{Re}T$ (here for simplicity we consider a single modulus T) from which the gauge coupling on the hidden wall can be expressed as $\alpha_h^{-1} = \alpha_{GUT}^{-1} - 2\epsilon \text{Re}T$. Clearly this choice reveals the critical upper bound on $\text{Re}T$ and the scale of the 5th dimension $\pi\rho$ (for a fixed value of M_{GUT}) if we demand that the hidden sector gauge coupling remain finite [3]. Although the exact value of this critical radius depends on the size of the threshold coefficients ϵ in the models we have considered in this paper there will be rough agreement of $\pi\rho$ for the choice $M_{GUT} = 2 \times 10^{16} \text{GeV}$. The other case one might have (which is realized in models 1, 4 and 9 of the Z_4 orbifolds listed in table 2) is that the stronger coupled sector contains the observable sector in which case we have the GUT scale relation $\alpha_{GUT}^{-1} = \text{Re}S - \epsilon \text{Re}T$ and $\alpha_h^{-1} = \alpha_{GUT}^{-1} + 2\epsilon \text{Re}T$. Thus there is no critical radius in this picture, if we imagine α_{GUT} as fixed. In both cases however, $\text{Re}T$ is determined by the requirement of obtaining the correct value of M_{pl} once α_{GUT} and M_{GUT} are fixed. But one can imagine (as in [8]) lowering the value of M_{GUT} from $2 \times 10^{16} \text{GeV}$, which would mean obtaining something other than the minimal supersymmetric extension of the standard model in $d = 4$. By lowering M_{GUT} (or in other words increasing κ) it would seem we can make $\pi\rho$ larger by not only the increase in κ but also of $\text{Re}T$ through the relation eq(26). However if we want to include the mechanism of hidden wall gaugino condensation to break supersymmetry as we have discussed in this paper, then this provided additional bounds on the maximum values of $\pi\rho$. This follows from the fact that if the wall on which condensation occurs is the more weakly coupled, then lowering M_{GUT} forces the hidden sector gauge coupling at M_{GUT} to smaller values and consequently it requires a longer renormalization group running (for a given value of b_h) in order to trigger condensation. This fact coupled with the lowering of M_{GUT} both point to a lowering of the scale Λ at which the gaugino's condense. We have seen in this paper that for the models considered we already obtain values of Λ around the optimal value of $10^{12} - 10^{13} \text{GeV}$ for gravity mediated soft supersymmetry breaking with $M_{GUT} = 2 \times 10^{16} \text{GeV}$. A natural lower bound on M_{GUT} would then be that which gives $\Lambda \approx 10^5 \text{GeV}$ which would correspond to gauge mediated soft supersymmetry breaking. From our discussions above this provides an upper bound on $\pi\rho$.

Let us state the results. In the $Z_2 \times Z_2$ case, and model 1, taking the direction $\text{Re}T^1 = \text{Re}T^2 = a \text{Re}T^3 = a \text{Re}T$ (with the dynamically favoured value $a = 1$ as discussed earlier) we find the critical value $\text{Re}T_c \approx 100$ for $\alpha_{GUT} = 0.04$ so that the physical critical radius $\rho_c = 16(2\alpha_{GUT})^{1/6} M_{GUT}^{-1}$. The actual value of $\text{Re}T$ obtained from the constraint eq(29), with above value of α_{GUT} and $M_{GUT} = 2 \times 10^{16} \text{GeV}$ is $\text{Re}T \approx 78$ which is close to the critical value. Because of this proximity and the fact that $\text{Re}T$ is very sensitive to M_{GUT} and Λ decreasing exponentially fast with T infact we cannot reduce M_{GUT} by much more than a factor of 2 or so. This implies the upper bound on the physical length ρ is only a few times that of the critical radius ρ_c given above. In model 3 $\text{Re}T_c \approx 50$ for the MSSM values of α_{GUT}, M_{GUT} . However we have seen that at these values $\text{Re}T \approx 78$ and we are in a forbidden region. Infact to remedy this one can either decrease α_{GUT} slightly or increase M_{GUT} to avoid this problem. For example in the plot of Λ in this case (Fig. 2) we have taken $\alpha_{GUT} > 0.02$.

In the Z_4 case there are two possibilities: either strong or weak wall condensation may occur. In the former, the analysis follows as above but now critical values of T are $\text{Re} = 160$ or $\text{Re} = 480$ depending on whether the particular $N = 1$ gauge group originates from either

Model	Gauge Group	Condensing Sector \tilde{G}	$b_{\tilde{G}}$	(n_1, n_2, n_3)
1	$E_6 \times U_1^2 \times SO'_8 \times SO'_8$	E_6	-18	(4, 4, 4)
2	$E_6 \times U_1^2 \times SO'_{16}$	none	—	(4, 4, -12)
3	$E_7 \times SU_2 \times SU'_8 \times U'_1$	E_7	-42	(4, 4, 12)
4	$SO_{12} \times SU_2 \times \tilde{SU}_2 \times SU'_8 \times U'_1$	none	—	(4, 4, -4)

Table 1: $N = 1$, $Z_2 \times Z_2$ orbifolds with nonstandard embeddings

the type A or B $N =$ gauge group discussed earlier. These allow for slightly smaller reductions in M_{GUT} before $\text{Re}T$ reaches its critical value than in the $Z_2 \times Z_2$ case being typically 1/5 and 1/10 times the MSSM value of 2×10^{16} GeV. Potentially the more interesting case is weak wall condensation where the critical radius is absent, and it would seem large values of ρ are possible. However the bounds that $\Lambda \geq 10^5$ GeV together with the observation that Λ decreases exponentially with T which is itself varies as $(M_{Pl}/M_{GUT})^2$ mean that M_{GUT} cannot be lowered significantly. In models 1 and 9 for example, we typically can only lower by a factor of 1/2 or 1/3 wrt to the MSSM value, and hence the maximum values of the radius ρ are again only a few times the critical values.

It is clear that since the magnitude of the $N = 1$ beta function in the condensing sector can never be too large (it is -90 for unbroken E_8) this restriction on the lowering of M_{GUT} and the consequent increase of ρ is pretty strong and fairly generic. One possible means around this would be if one could find threshold corrections involving several T_i -moduli where the instanton numbers n_i have opposite signs for at least two values of the index i . Then one can imagine that the above lower bound on M_{GUT} could be weakened if partial cancellation occurs amongst the moduli in the threshold corrections, as we lower M_{GUT} and hence increase $\text{Re}T_i$. It would be interesting to find orbifold models where this happens and to estimate the maximum allowed value of ρ which might be orders of magnitude larger than ρ_c .

As noted at the end of Section 2, it is straightforward to construct models with two or three condensates using the basis models from tables 1 and 2. To achieve this, one needs to identify suitable F- and D-flat directions in the effective field theoretical Lagrangian. Then, typically, nonvanishing vevs of scalar fields along these flat directions can be chosen in such a way, that there remain additional nonabelian sectors with negative beta function coefficients, thus providing additional condensates. This can happen for instance to one of the SO'_8 factors in the model 1 from table 1, or to the E_6 factor in the model 1 of table 2. Detailed study of these models requires identification of the Yukawa couplings, i.e. of the perturbative superpotential, for all the massless states. This is doable, however complete analysis of this type lies beyond the framework of the present paper. Preliminary considerations support clearly the scenario outlined in Section 2.

Model	Gauge Group	Condensing Sector \tilde{G}	$b_{\tilde{G}}$	Λ (in units of M_{Pl})	n_3
1	$E_6 \times SU_2 \times E'_7 \times SU'_2$	E'_7	-40	8×10^{-5}	-12
2	$E_6 \times SU_2 \times SO'_{16}$	SO'_{16}	-10	2×10^{-5}	4
3	$SO_{14} \times E'_7$	E'_7	-42	8×10^{-5}	4
4	$SO_{14} \times SO'_{12} \times SU'_2$	SO_{14}	-2	$\leq 10^{-10}$	4
5	$SO_{10} \times SU_4 \times E'_7$	E'_7	-42	7×10^{-5}	4
6	$SO_{10} \times SU_4 \times SO'_{12} \times SU'_2$	SO'_{12}	-14	10^{-7}	4
7	$SU_8 \times SU_2 \times E'_8$	E'_8	-90	3×10^{-3}	-12
8	$SU_8 \times SU_2 \times E'_7 \times SU'_2$	E'_7	-42	6×10^{-5}	4
9	$SU_8 \times SU_2 \times SO'_{16}$	SO'_{16}	-26	1.5×10^{-5}	4
10	$SU_8 \times E'_6 \times SU'_2$	E'_6	-18	10^{-6}	4

Table 2: $N = 1, Z_4$ orbifolds with nonstandard embeddings (U_1 factors suppressed)

4 Soft Terms

In this section we want to consider typical expressions for soft supersymmetry breaking operators that are induced due to the presence of a gaugino condensates on either of the walls, or on both of them. This mechanism is naturally described within a $5d$ framework where transmission of supersymmetry breaking between walls occurs through bulk field interactions with the walls. Although the framework of $5d$ supergravity is well established, there are subtleties that arise in the H-W formalism when reducing to $d = 4$ in the presence of condensates, which were discussed in [24]. There, it was argued that a method of reduction from $d = 5$ to $d = 4$ that captures the dynamics of the H-W picture, is to integrate out (via their equations of motion) the bulk fields that couple to condensates on (in general) both walls. In this way one obtains the following set of soft operators in the case that condensates Λ_i , $i = 1, 2 = (+, -)$ appear on opposite walls [24]

- **Gravitino Mass**

$$m_{3/2}^2 = \frac{(S + \bar{S})^{2/3}}{2^{2/3} 12 M^4} (\Lambda_1^3 + \Lambda_2^3)^2 + \frac{(T + \bar{T})^2}{432 M^4} \epsilon^2 (\Lambda_1^3 - \Lambda_2^3)^2 \quad (35)$$

where M is the reduced $d = 4$ Planck mass

- **Tan(θ)**

$$\tan(\theta) = \sqrt{3} \epsilon^{-1} \frac{S + \bar{S}}{T + \bar{T}} \frac{\Lambda_1^3 + \Lambda_2^3}{\Lambda_1^3 - \Lambda_2^3} \quad (36)$$

- **Gaugino Masses**

Here there is in general, a splitting between the gaugino masses arising on different walls,

which we denote by $M_{1/2}^\pm$

$$M_{1/2}^\pm = \frac{\sqrt{3}m_{3/2}}{(S + \bar{S}) \pm \epsilon(T + \bar{T})} \left(\sin(\theta)(S + \bar{S}) \pm \epsilon \cos(\theta) \frac{(T + \bar{T})}{\sqrt{3}} \right) \quad (37)$$

- **Trilinear Scalar Interactions**

Assuming vanishing CP-violating phases then generic A^\pm -term takes the form

$$A^\pm = \sqrt{3} \ m_{3/2} \left(\sin(\theta) \left(-1 \pm \epsilon \frac{3(T + \bar{T})}{3(S + \bar{S}) \pm \epsilon(T + \bar{T})} \right) + \sqrt{3} \cos(\theta) \left(-1 + \frac{3(T + \bar{T})}{3(S + \bar{S}) \pm \epsilon(T + \bar{T})} \right) \right) \quad (38)$$

- **Chiral Scalar Masses**

The chiral scalar masses $m_{i\bar{j}}^{2\pm}$ are found to be

$$m_{i\bar{j}}^{2\pm} = K_{i\bar{j}} \left(m_{3/2}^2 - \frac{3m_{3/2}^2}{3(S + \bar{S}) \pm \epsilon(T + \bar{T})} \left(\pm \epsilon(T + \bar{T}) \left(2 - \frac{\pm \epsilon(T + \bar{T})}{3(S + \bar{S}) \pm \epsilon(T + \bar{T})} \right) \sin^2 \theta + (S + \bar{S}) \left(2 - \frac{3(S + \bar{S})}{3(S + \bar{S}) \pm \epsilon(T + \bar{T})} \right) \cos^2 \theta - \frac{\pm \epsilon 2\sqrt{3}(T + \bar{T})(S + \bar{S})}{3(S + \bar{S}) \pm \epsilon(T + \bar{T})} \sin \theta \cos \theta \right) \right). \quad (39)$$

If we now consider the single condensate case (e.g. $\Lambda_2 = 0$) then it is clear from the above expressions that Λ_1 sets the scale for the soft terms, as well as the appearance of reduced Planck mass M in the expression for gravitino masses. It is interesting to see the dependence of the various soft terms on ϵ (which corresponds to a choice of nonstandard embedding), in the case where we take standard model values for α_{GUT} and M_{GUT} , and eliminate ReS, ReT using the earlier definitions and the fit to M_{pl} in $d = 4$. Although applying this procedure gives soft terms that depend on ϵ , plotting these in a model independent fashion is problematic, since we know that the beta function coefficients in Λ_1 change as ϵ varies. A better solution is to normalize the plots by dividing the soft term by appropriate powers of Λ_1 and M (effectively yielding dimensionless expressions). This is what we have done in Figures 5-7⁹ (in the case of chiral scalar masses we plot the diagonal combination $K^{-1ij} m_{j\bar{k}}^{2\pm} M^4 / \Lambda_1^3$). To get absolute normalization of soft terms in a given model requires the data from the plots together with the value of Λ_1 in the model.

The striking feature of the pattern of the soft breaking operators is the strongly marked asymmetry between soft terms for fields stemming from different walls, both for small and large values of the $\tan(\theta)$. It is straightforward to imagine the Standard Model group to be a mixture of the gauge factors originally¹⁰ contained in both E_8 and E'_8 living on different walls (at least the $U(1)$ factor(s) can easily be such mixtures), see [6]. In such a case this asymmetry would be a clear low energy signature of the ‘double-wall’ structure at high energies.

⁹Note, that we assume the convention that θ varies between $-\pi/2$ and $+\pi/2$.

¹⁰i.e. before turning on the vacuum bundle.

5 Conclusions

In this paper we have focussed on the viability of the mechanism of gaugino condensation to generate supersymmetry breaking at acceptable energy scales in strongly coupled $E_8 \times E_8$ heterotic string theory. In this context we have considered compactifying the boundary theories to $d = 4$ on orbifolds, in order to have explicit strongly coupled string models as the framework to consider the 4d dynamics of the hidden sector condensation.

We have concluded that if two or three condensates are switched on *different* walls, then supersymmetry is always broken. If a larger number of condensates are switched on, then one obtains solutions with unbroken supersymmetry, but, possibly, stabilizing 4d moduli. From the five dimensional point of view, the racetracks on the walls are able to fix boundary expectation values of the 5d moduli, thus giving rise to the scenario described recently in [26].

In all the explicit examples we have studied the scale of condensation Λ is smaller than the scales associated with either the 5th or orbifold compact dimensions and so justifies the treatment of the dynamics of the condensate in the 4d framework. We have seen that phenomenologically reasonable values of $\Lambda \approx 10^{-5} M_{pl}$ are obtainable in a number of models with the MSSM value of M_{GUT} and α_{GUT} . In the case of threshold corrections which depend only on a single T modulus, this result is particularly interesting as such a modulus is determined in terms of M_{pl} , M_{GUT} and α_{GUT} by the requirement that the correct value of M_{pl} emerge in $d = 4$. Thus it seems that the M-theory thresholds are such that they appear to favour MSSM values of M_{GUT} in as much as if we try to lower significantly the value of M_{GUT} from this, then Λ is lowered exponentially quickly, due to the sensitivity of $\text{Re}T$ to M_{GUT} . The same is roughly the case when several moduli are present as we have seen in the $Z_2 \times Z_2$ orbifold examples. It would seem that to accommodate smaller values of M_{GUT} within models that also generate hierarchical hidden wall condensation requires some partial cancellation of moduli field expectation values in the threshold corrections on that wall, as we discussed above.

Even though as we have stated, the magnitude of Λ justifies a 4d renormalizable field theory picture, the actual supersymmetry breaking in the observable sector appears through soft terms generated by certain 5d bulk fields that couple to the condensates. What we have seen in some of the models studied here is that supersymmetry breaking source terms that arise from gaugino condensation have approximately the right hierarchical mass scale to generate TeV scale supersymmetry breaking in the visible sector. To visualize some of the nontrivial features of this supersymmetry breaking mechanism in the presence of two gauge sectors with different gauge kinetic functions, we have plotted the soft susy breaking operators for fields living on different walls as a function of the parameter ϵ which controls the nonuniversal part of the gauge kinetic function. Different values of the ϵ correspond to different embeddings. There is a noteworthy asymmetry between soft terms on different walls, which might offer a low energy signature of the ‘double wall’ structure of the fundamental theory.

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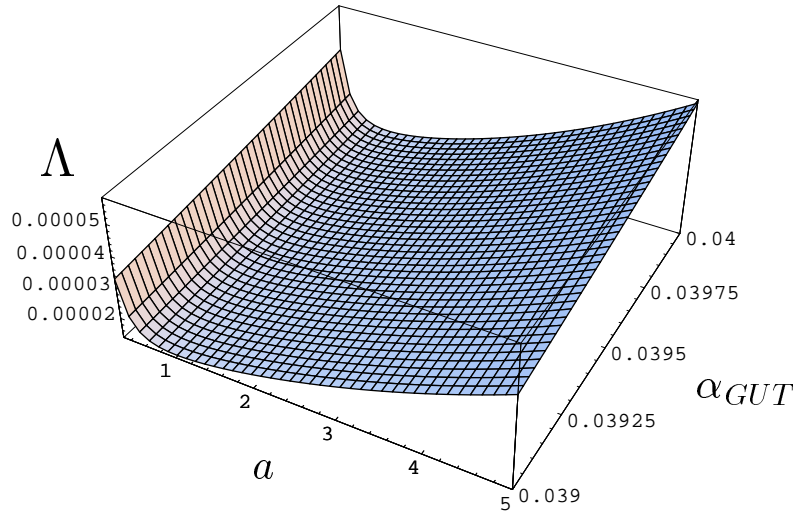


Figure 1: Model 1 : Λ as a function of a and α_{GUT} .

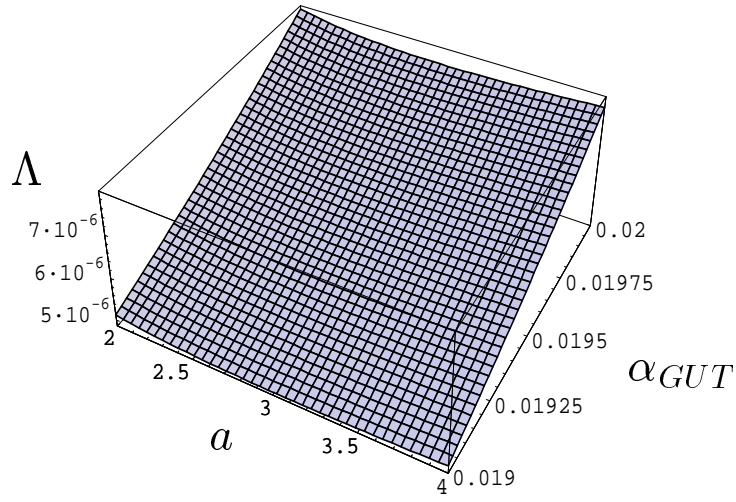


Figure 2: Model 3: Λ as a function of a and α_{GUT} .

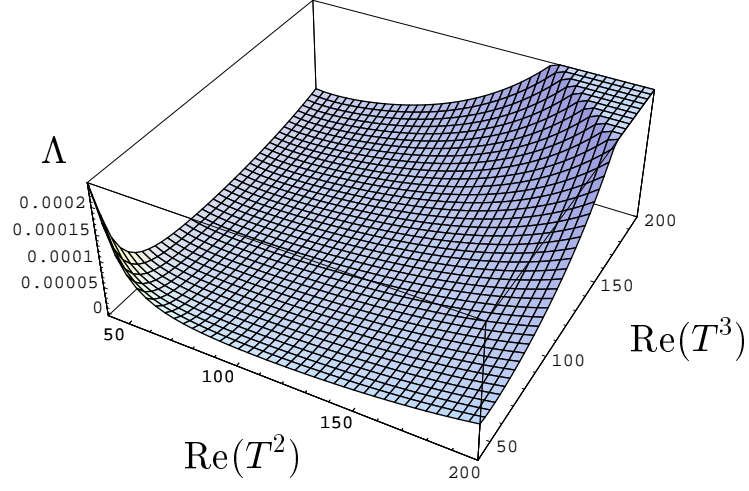


Figure 3: Model 1: Global minimum of Λ as a function of $\text{Re}(T^2)$ and $\text{Re}(T^3)$.

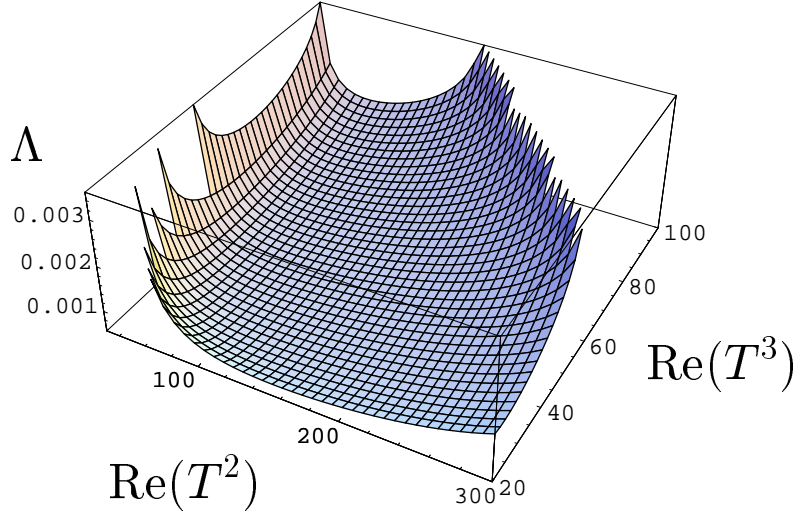


Figure 4: Model 3: Global minimum of Λ as a function of $\text{Re}(T^2)$ and $\text{Re}(T^3)$.

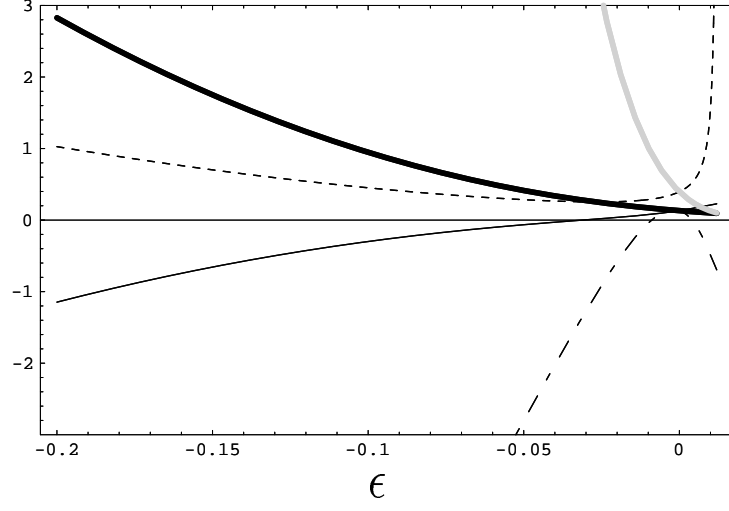


Figure 5: $m_{3/2}^2$ (bold black), $M_{1/2}^{2\pm}$ (dotted/grey solid curves) and $m_{ij}^{2\pm}$ (dashed/black solid curves) vs ϵ .

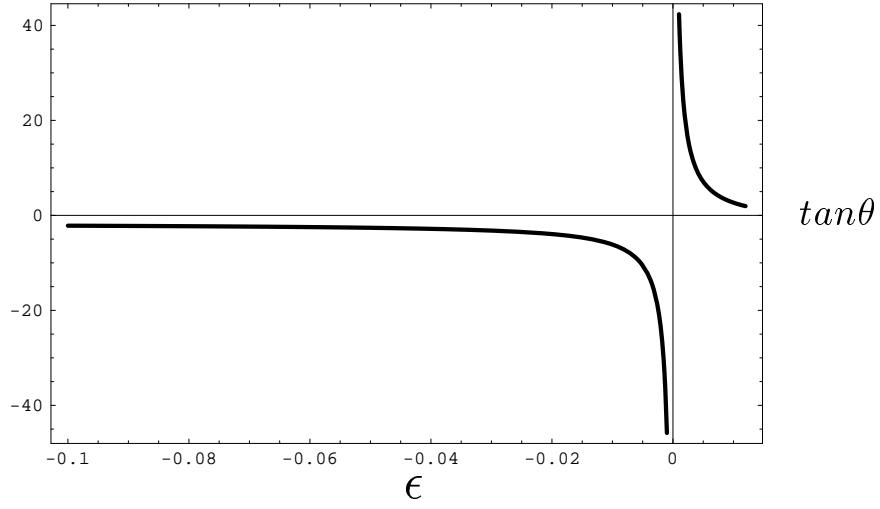


Figure 6: $\tan(\theta)$ vs ϵ .

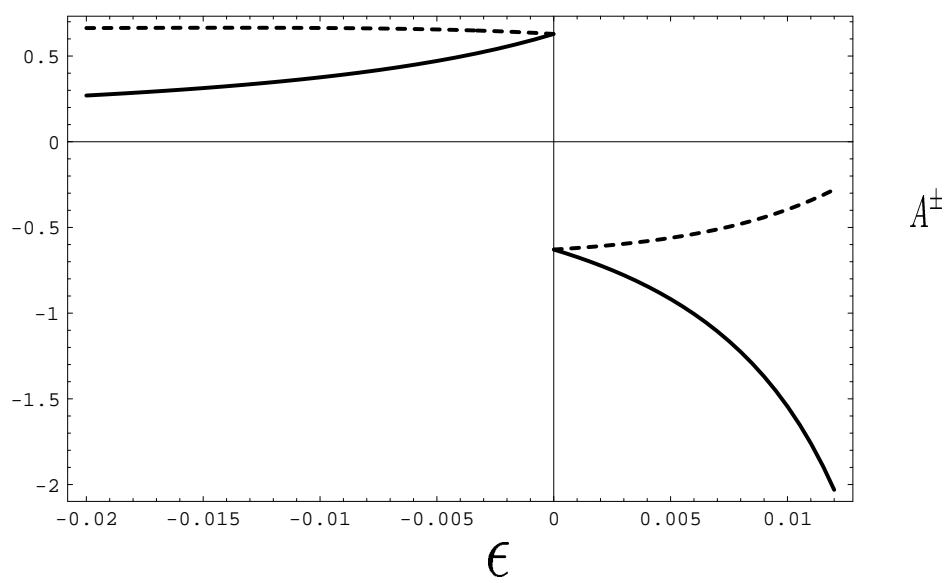


Figure 7: A^\pm (dashed/solid curves) vs ϵ .